

# OPTIMIZATION OF A VEHICLE MIX UNDER ENVIRONMENTAL AND ECONOMICAL CONSTRAINTS

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## ABSTRACT

Car manufacturers face now three types of constraints when they want to adopt a new technological configuration of a vehicle: environmental constraints, economical constraints and legislation constraints. In this context, Renault company wants to forecast, in a given car segment, what its customer offer might be within the next five to ten years. The main engine technologies (or technological tracks) are envisaged and assessed regarding their performances and respect of constraints. We present in this paper a two-stages decision-making process to define what the best strategy in term of a future vehicle mix (of technologies) on the market (for a given car segment) is. The first stage of our decision-making process is a multicriteria analysis of possible technologies from the viewpoints of the customers and the Renault company itself. The technologies will be ranked from both viewpoints and only a subset of these technologies (the best tradeoffs) are considered as interesting to be proposed on the market and then kept as inputs of the second stage. The PROMETHEE decision-making method is used to compare the technologies through their scores under five criteria. The second stage of our decision-making process is a multi-objective optimisation of the vehicle mix strategy. First, one proceeds to an exhaustive enumeration (under the global constraints of market sale foreknowledge and average CO<sub>2</sub> emission) of all possible vehicle mix configurations. This exhaustive enumeration of mix configurations is performed through integer constraint programming techniques. Second, these solutions are considered under the two following objectives: maximizing the profit for Renault and minimizing the consumption of natural resources. Then, the subset of optimal Pareto solutions is determined and the Pareto solutions are graphically displayed on a two-dimensional representation. The final choice of a preferred solution of a vehicle mix is then commented. This two-stages decision-making technique may be applied to other issues of technology mixes subject to global constraints.

*Keywords: Multicriteria decision making, multiobjective optimization, technological choice, environmental constraints, tradeoffs, Pareto solutions, constraint programming, PROMETHEE*

## 1 INTRODUCTION

Car manufacturers face now three types of constraints when they want to adopt a new technological configuration of a vehicle:

- Environmental constraints:
  - Lower the environmental impacts of the vehicle (CO<sub>2</sub> emissions, use of natural resources such as metal or plastics)
  - Meeting ever growing customers' green expectations (hybrid configurations, flex-fuel systems)
- Economical constraints:
  - Limiting overcosts due to new technologies (for the car manufacturer and for the customers as well)
- Legislation constraints:
  - Respect legislation thresholds or self-objectives the car manufacturer has defined (e.g.: Renault aims at obtaining an average CO<sub>2</sub> emission rate less than 140g/km by

2008 on its car offer)

- Fiscal incentives of governments (for purchasing hybrid cars for instance) and insurances

In this context, Renault company wants to forecast, in a given car segment, what its customer offer might be within the next five to ten years. The main technologies (or technological tracks) are already known, for mentioning some of them corresponding to identified design choices and development scenarios: *Mild Hybrid, Strong Hybrid, Medium Downsizing, Strong Downsizing, Medium Weight, Strong Weight...*

We present in this paper a two-stages decision-making process to define what the best strategy in term of a future vehicle mix (of technologies) on the market (for a given car segment) is.

The first stage of our decision-making process is a multicriteria analysis of possible technologies from the viewpoints of the customers and the Renault company itself. The technologies will be ranked from both viewpoints and only a subset of these technologies (the best tradeoffs) are considered as interesting to be proposed on the market and then kept as inputs of the second stage. Five decision criteria are considered: *purchasing price, fuel consumption, fiscal incentives, CO2 emission rate and natural resources consumption*. The PROMETHEE decision-making method is used to compare the technologies through their scores under the five criteria. We used conventional linear preference functions with two thresholds for quantifying the *preference* and the *indifference* limits between two scores. These thresholds have been identified for each criterion and for the customers on the one side (data extracted from an existing market study) and for Renault company on the other side (experts group workshops). Preference flows and ranking graphs are then established for judging the degrees of dominance of a given technology over another one. Two ordered graphs have been obtained from the customers and Renault viewpoints. Despite some order discrepancies, the most deserving technologies clearly appear.

The second stage of our decision-making process is a multi-objective optimization of the vehicle mix strategy (or combination).

First, one proceeds to an exhaustive enumeration (under constraints) of all possible vehicle mix configurations. An overall sale foreknowledge is considered within which three elected technologies must share this total number of vehicles. A second global constraint is imposed to the mix: the respect of the average CO2 emission rate of 120g/kg. The numbers of vehicles under each of the three technologies are considered as three decision variables which are made discrete in considering a basic batch size increment. The exhaustive enumeration of mix configurations is performed through integer constraint programming techniques.

Second, these solutions are considered under the two following objectives:

- Maximizing the profit for Renault,
- Minimizing the consumption of natural resources.

Then, the subset of optimal Pareto solutions is determined and the Pareto solutions are graphically displayed on a two-dimensional representation. The final choice of a preferred solution of a vehicle mix is commented.

In conclusion, we evoke the interest this approach has provoked within the Renault department of financial simulations and beyond. Such an approach can be generalized for other studies like for instance assessing the benefits of recycling strategies, provided the data extracted from the customers and Renault expert preferences are valid.

## **2 FIRST STAGE: MULTICRITERIA ANALYSIS OF CAR TECHNOLOGIES**

### **2.1 MCDA methods: the choice of PROMETHEE-GAIA**

Multi-Criteria Decision Analysis (MCDA) consists in ranking and/or rating a limited number of alternatives (actions, projects, products, technologies...) under a given number of determining criteria (see [1, 2] for a general presentation). All the methods consider as an input a decision matrix made of columns of assessed scores for the considered alternatives, each row corresponding to the determining criteria.

One generally distinguishes between two families of approaches:

- The *Multi-Attribute Utility Theory* (MAUT) for which the basic axiom is that there exists an overall utility function aggregating all the criteria scores. Different theories and models of aggregation coexist (see [3-7]).
- The *outranking methods* which do not assume any preexisting preference aggregation model. They are based on dominance relations between the scores of different alternatives, and on possible deductions that can be performed from that relational network (see [8-10]).

PROMETHEE, standing for Preference Ranking Organization METHod for Enrichment Evaluations, is a famous outranking method which has been developed by professors J.P. Brans and B. Mareschal of Ecole Polytechnique de Bruxelles (see [9]). We have used the commercial version of PROMETHEE by Visual Decision company<sup>1</sup>, named Decision Lab 2000. This user-friendly version is named PROMETHEE-GAIA since a set of graphical tools named GAIA, standing for Graphical Analysis for Interactive Assistance, permits a smart graphical display of the output results.

## 2.2 Principles of preferences between scores and preference flows

Let us imagine two alternative cars (named *popular* and *classy*) that we compare under five determining and conventional criteria: *price*, *power*, *fuel consumption*, *space* and *comfort*. This example is adapted from one of Brans, Mareschal and Vincke [9]. An outranking method like PROMETHEE-GAIA assumes that a decision maker tends to pairwise comparing the alternatives under any criterion before deciding which alternative is globally preferred. Figure 1 illustrates the input decision matrix (the three median columns) containing the assessed criteria scores. In pairwise comparing these scores, one can summarize in two “*difference*” columns the advantageous *differences* for one of the two alternatives, knowing that *price* must be lowered as much as possible, *power* must be maximized, and so on.... A third stage of data preparation is to be able to quantify if these score differences are significant in the consumer judgment. Is he/she sensitive to this difference: not at all, very much or partially? Does it exist a *preference threshold* beyond which there is undoubtedly a dominance of a score over the other one? Does it exist an *indifference threshold* below which the customer is not sensitive to the scores difference? How does the preference value evolve between both thresholds? The PROMETHEE-GAIA method proposes to associate a *preference threshold*, an *indifference threshold* and a preference function to each decision criterion. With these data, preference values can be quantified between 0 and 1 (see the two extreme columns of Table 1), 0 whenever the score difference is below the *indifference threshold*, 1 whenever the score difference is beyond the *preference threshold*.

Table1: Example of a decision matrix for two alternative cars (adapted from [9])

Preference	Difference	Decision matrix			Difference	Preference
1.0	- 22,000 €	15,000 €	<i>Price</i>	38,000 €		0.0
0.0		50 kW	<i>Power</i>	90 kW	+ 40 kW	1.0
0.5	- 1 L/100km	7.5 L/100km	<i>Fuel consumption</i>	8.5 L/100km		0.0
0.0		Bad	<i>Space</i>	Good	+ 2 points	0.5
0.0		Very bad	<i>Comfort</i>	Very good	+ 4 points	1.0
<i>Popular car</i>			Criteria	<i>Classy car</i>		

Finally, an *aggregate preference degree* may be computed for each alternative by a weighted average of the elementary preferences. Let us imagine that all the criteria weights are equal, the *aggregate preference degrees* would be of 0.3 for *popular* car and 0.5 for *classy* car.

<sup>1</sup> [http://www.visualdecision.com/dlab\\_f.htm](http://www.visualdecision.com/dlab_f.htm)

### 2.3 Data preparation stage on our case study

In our decision context (see chapter 1), one would like to rank a set of car technology alternatives (especially engine technologies) for a good compromise under economical, environmental and legal criteria.

The **technology alternatives** have been easily enumerated by experts as: *Mild Hybrid, Strong Hybrid, Medium Downsizing, Strong Downsizing, Medium Weight, Strong Weight...* Let us just say that hybrid technologies are costly for an expected lowering of fuel consumption; they also benefit from strong fiscal incentives. *Downsizing* technologies consist in reducing the size of engines by increasing the internal pressure and then the thermodynamic efficiency; in addition, there will be a substantial lowering of natural resource consumption. *Weight* technologies simply consist in choosing lighter materials and in lowering the number of parts which results in a substantial lowering of fuel consumption and a strong lowering of natural resource consumption. Two application levels are considered for the three families of car technologies.

The **decision criteria** have been more discussed by the expert group. Five criteria have been retained in respect of exhaustiveness and non-redundancy principles considering their influence on economical, environmental and legal expectations for the two main stakeholders: the customers on the one side and the car manufacturer (here, Renault SA company) on the other side. In fact, a consumer is and will still stay sensitive (in the next future) to the car *price*, the *fuel consumption*, *fiscal incentives*, and more and more to the *CO<sub>2</sub> emission* rate. From the Renault side, the *price* is linked to the expected margin, and the *CO<sub>2</sub> emission* rate and the *natural resource consumption* are preoccupying performances from at least a legal point of view and also a marketing differentiation strategy. Other criteria could be added to this primary list (not considered here) such as: *power, comfort, inhabitability, maintenance facility, warranty, reliability*, and *technology feasibility* since some technologies are not yet fully under control like: *Strong Hybrid, Strong Downsizing* and *Strong Weight*. Measurement scales and a direction of optimization (expected minimization or expected maximization) is also precised for the five decision criteria. All these information are summarized in Table 2.

Table 2: The 5 decision criteria

Criterion	Objective	Scale and unit	Min/Max
Price	Economical	Euros (€)	Minimize
Fiscal incentives	Economical	Euros (€)	Minimize
Fuel consumption	Economical	Euros (€)	Minimize
CO <sub>2</sub> emission	Environmental and legal	% lowering	Minimize
Natural resource consumption	Environmental and legal	kg	Minimize

The **scores** for the 6 alternative technologies have been assessed relatively to a reference solution which is a conventional fuel technology belonging to the same car segment. Table 3 presents relative improvements and even worsenings (always the case for the *price*) assessed with the defined units. These scores are factual and do not depend on any subjective interpretation from a stakeholder.

Conversely, the **indifference and preference thresholds** are dependent from that fact that the decision problem is considered from the customer or the car manufacturer viewpoints. Thresholds from the customer viewpoint are provided in Table 3. For reasons of confidentiality, the thresholds from the car manufacturer viewpoint are not provided.

The PROMETHEE method proposes 5 types of preference functions: *Linear, U-shape, V-shape, Gaussian* and *usual* (see [9, 10]). We have here chosen the *linear* type for the 5 criteria. The example of the *Linear* type preference function for the *price* criterion from the customer(s) viewpoint is provided in Figure 1.

Finally, the **criteria weights** are assessed via classical pairwise comparison techniques (see for instance [5, 7]). The weight vector for the customer(s) is presented in Figure 2.

All the data (thresholds and weights) concerning the customers have been extracted from actual market studies performed by Renault company.

Table 3: Table of score evaluations, criteria scales, criteria preference function types and thresholds

	PV	Fiscalité	Conso Fuel	Conso RN	CO2
Min/Max	Minimize	Minimize	Minimize	Minimize	Minimize
Weight	1.6000	0.7646	1.5384	0.3479	0.7491
Preference Function	Level	Level	Level	Level	Level
Indifference Threshold	500.0000	250.0000	250.0000	25.0000	5.0000
Preference Threshold	1000.0000	500.0000	500.0000	50.0000	10.0000
Gaussian Threshold	-	-	-	-	-
Threshold Unit	Absolute	Absolute	Absolute	Absolute	Absolute
Unit	EUROS	EUROS	EUROS	Kg	%
Med. Dwn	500	-250	-450	-58	-12
Str. Dwn	800	-250	-600	-65	-16
<b>Mild Hybrid</b>	3038	-250	-412	45	-8
<b>Full Hybrid</b>	8034	-2500	-937	130	-25
Med. Weight	2194	-250	-187	-157	-5
Str. Weight	6250	-250	-300	-240	-8

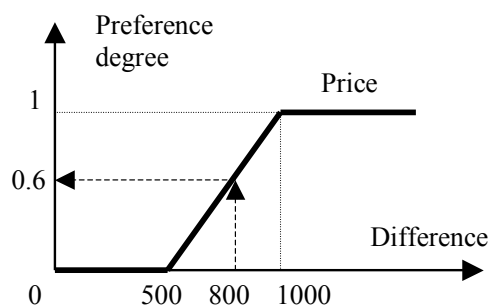


Figure 1: Example of the Linear type preference function for the price from the customer(s) viewpoint

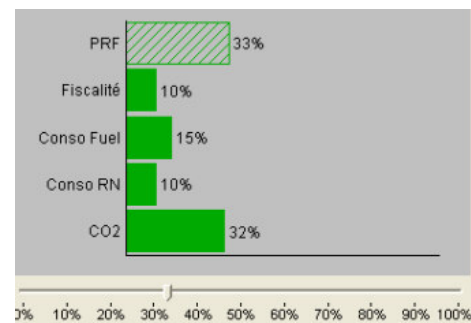


Figure 2: Weight vector for the customer

## 2.4 Decision making process

From the input data of Table 3, it is now possible to obtain all the preference evaluations between pairs of scores (under a given criterion and for two alternatives). Let us note  $A$  the set of technology alternatives, and  $a$  and  $b$  two given alternatives. As it has been above-explained, *aggregate preference degrees* are globally computed when comparing an alternative onto another. The *aggregate preference degree* of alternative  $a$  onto alternative  $b$  is given by the following formula:

$$\Pi(a,b) = \frac{1}{\sum_j w_j} \sum_i w_i \cdot P_i(a,b) \quad (1)$$

with  $P_i(a,b)$  being the preference value of  $a$  over  $b$  under the  $i^{\text{th}}$  criterion and  $w_i$  the weight of the  $i^{\text{th}}$  criterion (for a given stakeholder, not modelled here). By definition,  $\Pi(a,b)$  is comprised between 0 and 1. Let us define the *positive flow* or *force* of alternative  $a$  as the overall preference of  $a$  over other alternatives. The positive flow is obviously defined by formula (2):

$$\Phi^+(a) = \sum_{b \in A} \Pi(a,b) \quad (2)$$

The greater the positive flow, the better the considered alternative. Let us define the *negative flow* or *weakness* of alternative  $a$  as the overall preference by which the other alternatives are preferred over  $a$ . The negative flow is obviously defined by formula (3):

$$\Phi^-(a) = \sum_{b \in A} \Pi(b,a) \quad (3)$$

The smaller the negative flow, the better the considered alternative. Finally, the *net flow* is defined as the difference between the positive flow and the negative flow:

$$\Phi(a) = \Phi^+(a) - \Phi^-(a) \quad (4)$$

The greater the net flow, the better the considered alternative.

On the basis of positive and negative flows, the PROMETHEE I method establishes a partial ranking of alternatives in following the rules that:

- alternative  $a$  is *preferred to*  $b$  if :  $\Phi^+(a) \geq \Phi^+(b)$  and  $\Phi^-(a) \leq \Phi^-(b)$ , at least one of the inequality relations being strict,
- alternative  $a$  is *indifferent to*  $b$  if :  $\Phi^+(a) = \Phi^+(b)$  and  $\Phi^-(a) = \Phi^-(b)$ ,
- alternatives  $a$  and  $b$  being *incomparable* otherwise.

In some decision cases, incomparabilities are not interesting and a total ranking is preferred for a facilitated decision. The net flow is then used in the PROMETHEE II ranking; only *preference* and *indifference* situations then occurs depending on the strict dominance or the equality of the net flows.

All these calculations have been performed with Decision Lab 2000 and the results have been graphically displayed via the GAIA tools for both output results of PROMETHEE I and PROMETHEE II and both stakeholders: the customer(s) and the car manufacturer. The data are not provided here for reasons of confidentiality.

## 2.5 Output results with the GAIA plane and weight sensitivity analysis

A first output result is provided by the GAIA method which is based on a statistical analysis, particularly a *principal component analysis* (PCA) of net flow values, evaluated separately for each criterion. A *GAIA plane* is then built as the principal plane issued from the PCA analysis. The criteria are then represented as vectors originated from the origin point. Figure 3 represents the GAIA plane for the customer(s). Two parallel vectors in the same direction means that the two criteria are perfectly correlated within the alternative set. Two opposite vectors means that the two criteria are antagonist. The length of these vectors correspond to the sensitivity of the criterion on the global evaluation of an alternative. A short criterion vector means that all the alternatives present similar performances for this criterion; such a criterion should not play a significant role on the PROMETHEE rankings, whatever the weight allocated to this criterion. Therefore, a brief exam of the criteria vector locations is sufficient to determine the main tradeoffs the Renault decision makers will have to face. Here (see Figure 3), we can clearly identify three groups of correlated criteria :  $\{price\}$  at right and down,  $\{natural\ resource\ consumption\}$  at right and up,  $\{fiscal\ incentives, fuel\ consumption, CO2\ emission\ rate\}$  to the left. It is much satisfactory to observe that fiscal incentives are well appropriate since they

promote technologies suited to the two main societal objectives of saving the fossil energies and minimizing the climate change.

On the GAIA plane, the alternative technologies are also represented by punctual icons. The proximity of these points stress similar impact or performance/score profiles for the considered technologies. Conversely, a distance apparently important stresses major differences in performance/score profiles. Clusters of similar technologies may be detected. Here (see Figure 3), one can notice that *Strong Downsizing* and *Medium Downsizing* are close, highlighting the similarity of situations from the customer viewpoint. Conversely, the *Full Hybrid* and *Medium Weight* have opposite profiles. There is also an interesting mapping between the graphical representations of the criteria vectors and the alternative points. Indeed, the orthogonal projection of an alternative location onto the direction of a given criterion vector provides the net flow value. Consequently, this is possible to quickly determine the forces and weaknesses of the alternatives by successive projections onto the criteria vectors.

Lastly, the (criteria) weight vector is also graphically represented as a vector (ended in Figure 3 by a red disc in a downward location). Its orientation determines the most favored criteria and the most ignored criteria in the tradeoff. When the weight vector is modified by the user, this vector is the only graphical item of the GAIA plane that is modified. It can be observed in an interactive manner in the Decision Lab 2000 software; a so-called “walking weights” facility allows the user to study the sensitivity of the weight distribution to the partial and total ranking results provided by PROMETHEE I and II computations (see next chapter). This simulation tool permits to make a final robust decision.

An user must keep in mind that the GAIA plane is the best 2D representation of a multi-dimensional problem. A part of information is then lost in this approximation. The quality of approximation is provided by the  $\Delta$  parameter appearing at the bottom of the screen window of Figure 3. This parameter (comprised between 0 and 100%) measures the statistical significance of the 2D projection; here a value exceeding 90 % means that the quality of the 2D information is high and that interpretations from these data are reliable.

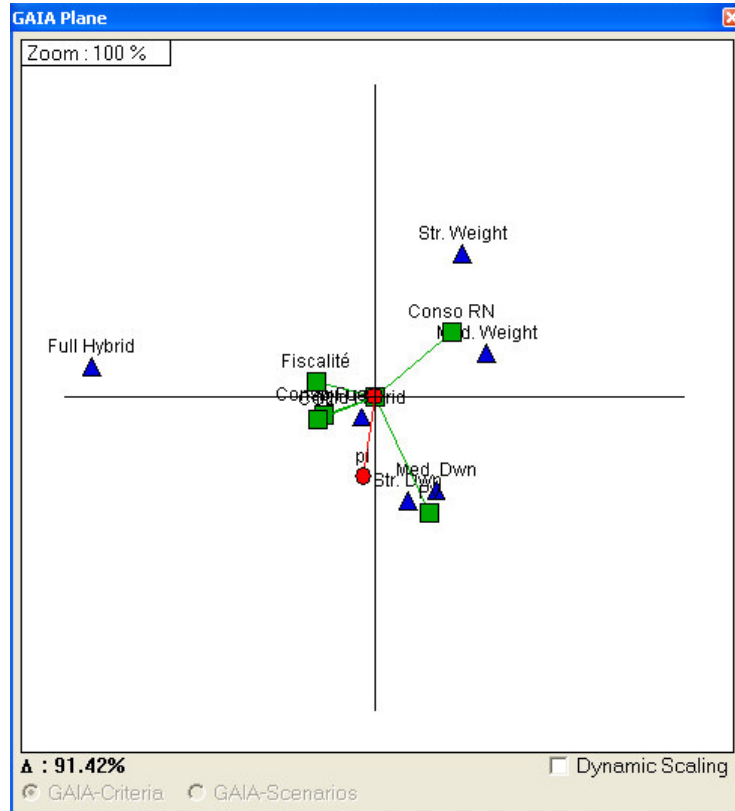


Figure 3: The GAIA plane for the customer(s)

## 2.6 Output results in terms of partial and total rankings

The output partial ranking of PROMETHEE I is given in Figure 4. Only the *Full Hybrid* solution is incomparable with other solutions since it has great forces but in the same time important weaknesses. Once totally ranked with the PROMETHEE II algorithm (see Figure 5), the *Full Hybrid* solution appears at the third rank for the customer(s).

For the customer(s), the final 3 preferred technology solutions are: first the *Strong Downsizing*, second the *Medium Downsizing* and third the *Full Hybrid*. Without providing much details for the Renault viewpoint, let us say that this final ranking stays unchanged.

Next, these 3 technologies are considered as interesting to be proposed on the market in a next future. They are then kept as inputs of the second stage of our two-stages decision making method, namely the multi-objective optimization of the vehicle mix strategy.

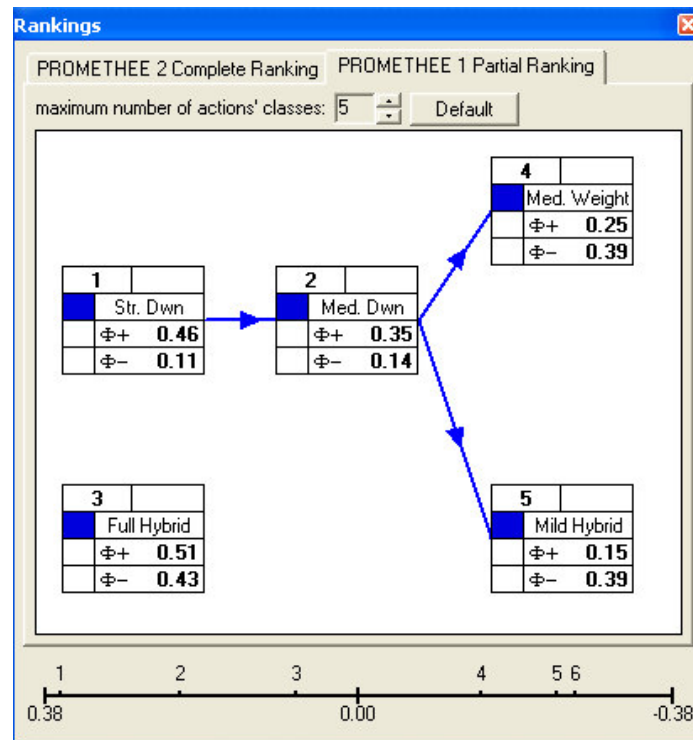


Figure 4: Partial ranking with PROMETHEE I for the customer(s)



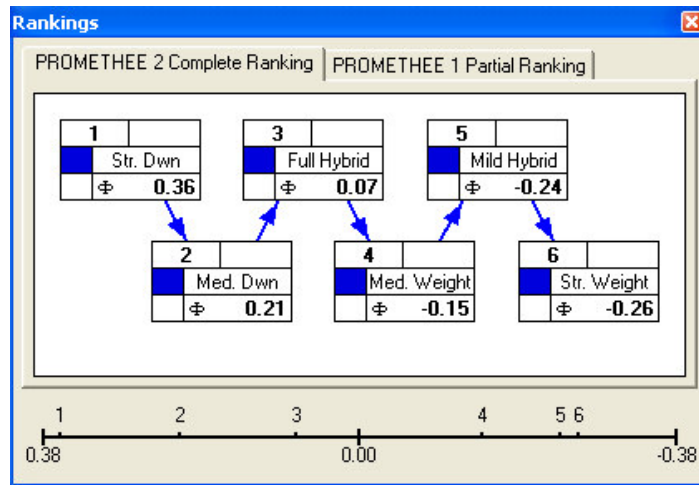


Figure 5: Total ranking with PROMETHEE II for the customer(s)

### 3 MULTI-OBJECTIVE OPTIMISATION OF THE VEHICLE MIX STRATEGY

#### 3.1 Modelling of the optimization problem

The issue for Renault is now to foresee the sales volumes the company should produce in a next future (a few years) for each of the 3 technologies. Let us consider this problem as a multi-objective optimization problem which can be expressed in successively considering the decision variables, the constraints and the objectives. The decision variables are  $x_1, x_2, x_3$  which represent the sales volumes (in car units) corresponding respectively to: *Medium Downsizing*, *Strong Downsizing* and *Strong Hybrid*. The constraints are threefold:

- **Sales:** The respect of the market sale foreknowledge of Renault at this time horizon. Here a Renault estimation at the targeted time horizon is 470,000 vehicle units (within the European Union) in the low car segment (including Twingo and Clio car models).
- **CO2:** The well-known (for Renault designers) ambitious “CAFÉ parameter” of 120g/km of CO2 average emission rate (i.e. a rate averaged on the vehicle mix sold on the market within the current year), which is a long term obligation fixed in 2012.
- **Price equity:** The purchasing overcost supported by the customers must be lower than the Renault benefits due to the technology.

Finally, we consider two objectives:

- **Margin:** Maximizing the Renault overall margin on the vehicle mix.
- **Natural Resources:** Minimizing the consumption of natural resources.

Of course, this modelling of the multi-objective optimization problem has required a long discussion process to converge. The mathematical and quantified expression of constraints and objectives require some additional data which are, for most of them, provided in Table 4. A mathematical representation of the optimization problem may then be:

$$\left\{ \begin{array}{l}
 (x_1, x_2, x_3) \in [0, 470000] \\
 \text{sales} \quad x_1 + x_2 + x_3 \leq 470000 \\
 \text{CO2} \quad 127x_1 + 121x_2 + 101x_3 \leq 120 \times 470000 \\
 \text{Price equity} \quad \begin{cases} -275x_1 \leq \text{Margin}_1 \\ -150x_2 \leq \text{Margin}_2 \\ 2438x_3 \leq \text{Margin}_3 \end{cases} \\
 \text{Maximize} \quad \text{Margin} = \text{Margin}_1 + \text{Margin}_2 + \text{Margin}_3 \\
 \text{Minimize} \quad \text{Natural\_Resources} = -58x_1 - 65x_2 + 130x_3
 \end{array} \right. \quad (5)$$

Of course, the calculation of the margins for the different sales volumes corresponding to the technologies are not detailed here; they depend non linearly on the volume variable  $x_i$  and on the estimated costs.

Table 4: Useful constants for expressing constraints and objectives

Nature of constraints and objectives →		Economical	Environmental	
Segment	Technologies	Price equity ( $\Delta$ client in €)	Natural resources (in Kg)	CO2 (in g/Km)
	Fuel (reference)	0	0	144.3
I	Medium Dwn	-275	-58	127
	Strong Dwn	-150	-65	121
	Strong Hybrid	2438	130	101

### 3.2 A new optimization algorithm based on Constraint Programming and Pareto set generation algorithms

The multi-objective optimization domain consists in optimizing several objectives in parallel (see [11] for a presentation). We have developed an original multi-objective optimization algorithm to solve this vehicle mix optimization problem. It is based on a primary discretization of the  $x_i$  values by steps of 10,000 vehicles. We are then in a discrete optimization problem of apparent size:  $48^3$  possible combinations of triplets.

Let us briefly recall that an optimal Pareto solution is a solution of a multi-objective optimization problem which is dominated by no other solution, a solution respecting the constraints. A dominated solution is a solution which is dominated on all the objectives by at least one solution. It exists many algorithms for generating sets of **optimal Pareto solutions** (see for instance [12]).

Our objective was to successively enumerate all the triplets which:

1. respect the 3 constraints in formula (5),
2. are not dominated by the current set of non-dominated Pareto solutions which have been already enumerated. In such a case (introduction of a new non-dominated Pareto set within the sub-list), the sub-list of current Pareto solutions is revised to eliminate the ancient solutions considered as non-dominated and which become dominated by the new optimal Pareto solution introduced.

For that purpose, we had to dynamically build a search tree in checking the constraints and eliminating dominated solutions. This has been done in using **Integer Constraint Programming (ICP)** techniques that we do not detail herein. However, let us say that ICP techniques have been developed in the seventies (see [13, 14]) for enumerating all the tuples solutions of a discrete mathematical problem and to find the best solution (in case of a mono-objective). We have re-programmed well known ICP *arc-consistency* techniques for dynamically checking that when running along the search tree, an inconsistency considering points 1 and 2 above-mentioned could be detected as soon as possible so as to straightforwardly prune out a large part of the combinatorial tree. One of the authors has already used such techniques to develop efficient enumeration techniques for space layout planning issues (see [15, 16]).

### 3.3 Optimization results

At the end of the algorithm, the list of optimal Pareto solutions corresponding to the discretization step is obtained. Among  $48^3$  possible solutions, only 159 combinations exist among which **5 are optimal Pareto solutions**. The 159 combinations stress a strongly constrained problem, and this is why the ICP technique used has been efficient to prune out very large parts of the combinatorial space (i.e. detecting quickly an inconsistency and then backtracking whereas being close to the root node of the search tree). The computation time was so short that it was almost instantaneous. If this computation

time had been important (it cannot be determined in advance), we would simply have increased the discretization step.

A graphical interface has been developed, especially to be able to model and optimize such vehicle mix optimization problems. Its main panel can be seen in Figure 6.

It can be noticed that Renault will not be able to sell more than 40,000 vehicles of the *Strong Hybrid* type since the constraint on price equity would not be respected. In the same manner, Renault will not be able to sell more than 43,000 vehicles of the *Medium Downsizing* type since the constraint on CO2 emission rate would not be respected.

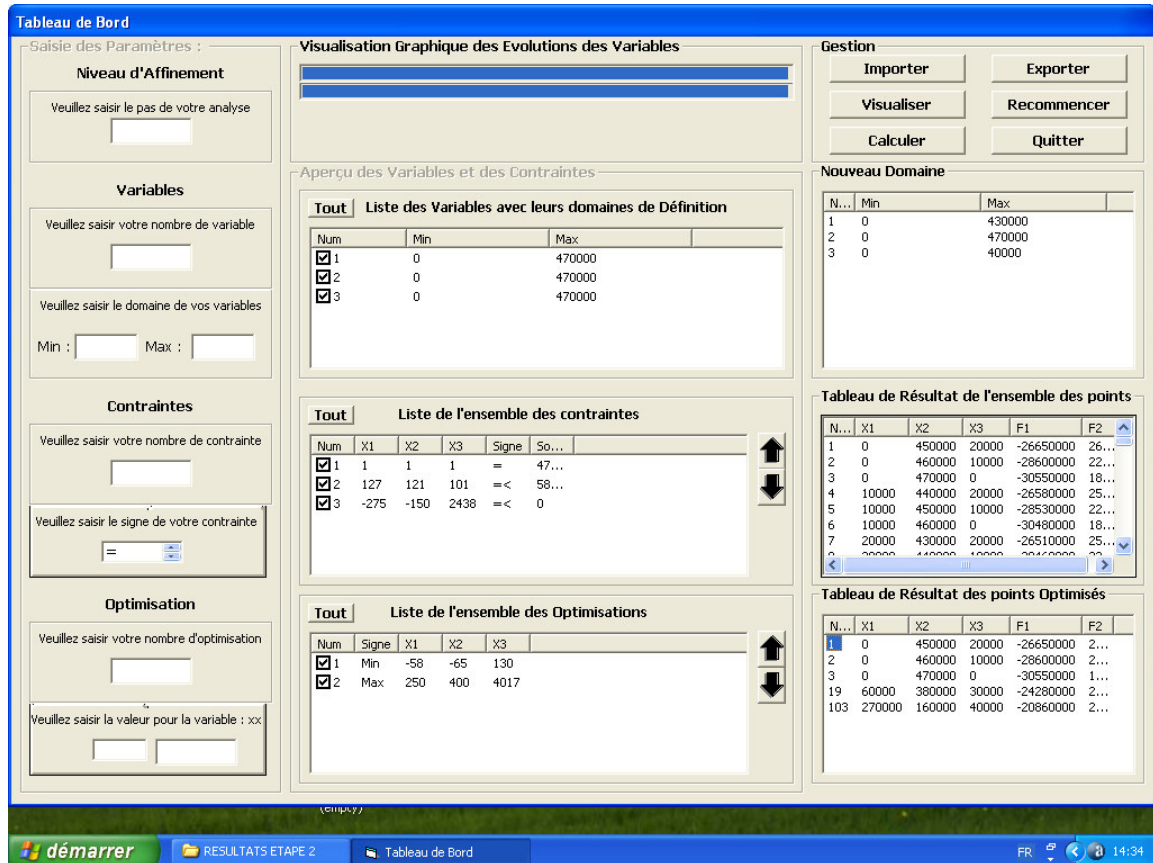


Figure 6: The graphical optimization interface developed for vehicle mix optimization problems

## CONCLUSION

This paper has presented a two-stages decision making method adapted to a classical decision making issue in car industry: a first technology choice of future promising car technologies, and a second optimization stage of the vehicle mix for a sub-list of the most promising car technologies. The first stage was a multicriteria analysis of 6 vehicle technologies via the use of PROMETHEE-GAIA method [9, 10] and from both customer and Renault viewpoints. The second stage was based on an original multi-objective optimization algorithm based on an Integer Constraint Programming technique and an embedded Pareto set construction. A complete and user-friendly graphical optimization interface has been developed for solving such vehicle mix optimization problems. It must also be noted that the data used for this optimization problem modeling are really non trivial and are of the best quality since they have been on the customer side measured through market studies and market sales foreknowledges, and on the Renault side extracted from an expert group during adapted workshops.

The methodology used and the 5 optimal solutions found (not detailed herein for reasons of confidentiality) have greatly convinced the Renault decision makers. They were willing to repeat this simulation to the others car segments.

Such a two-stages optimization methodology can be generalized for other studies. Within the Renault company, they envisage for instance to assess the benefits of a mix of targeted recycling strategies.

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