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COMBINING ANALYSIS OF DIFFERENT PERFORMANCES THROUGH THE USE OF DIMENSIONAL ANALYSIS

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ABSTRACT

This article demonstrates that it is possible and valuable to develop a generic evaluation approach for preliminary design based on dimensional analysis and a method for multiple evaluations of heterogeneous performances. The use of dimensional analysis theory allows building a minimal number of aggregate indicators based on the adequate combination of elementary performance variables of the same type. The proposed method is particularly convenient during the preliminary design stage. The reason for using dimensional analysis for early design purpose is that it is strongly based on concepts of similarity and analogy and that analogical reasoning has been proved to play a central role in design contexts. Mathematically, similarity refers to a transformation of variables that leads to a reduction in the number of independent variables that specify a problem. In addition, we argue that dimensional analysis can be integrated in the existing theoretical framework of design (i.e. General Design Theory). We have applied the approach to a case study of the design of a workplace by taking into consideration, for simplification purposes of this article, only two requirements: improving the visual comfort of the occupants and reducing the energy consumption level.
This approach may be used in many design contexts in providing a fruitful framework to structure performances and evaluation stage.

Keywords: Dimensional analysis, environmental metrics, exergy, cost analysis

1 INTRODUCTION

The purpose of our article is to demonstrate that it is possible and valuable to develop a generic evaluation approach for preliminary design that is based on dimensional analysis and a method for multiple evaluations of heterogeneous performances, in the sense of diverse natures. This is typically the case when dealing with sustainable development properties. In physics, the use of *dimensional analysis theory* allows minimizing the number of determining variables for explaining physical phenomena in adequately aggregated combinations of elementary performance variables, linked by a logic of measurement units. The proposed method is particularly convenient during the preliminary state of design. The reason to use dimensional analysis for early design purposes is that it is strongly based on concepts of similarity and analogy and that analogical reasoning has been proved to play a central role in design contexts [1] [2]. Similarity refers to some equivalence between two things or phenomena that are actually different. Mathematically, similarity refers to a transformation of variables that leads to a reduction in the number of independent variables that specify a problem. These two characteristics of dimensional analysis, when applied to the design process, may provide interesting breakthrough in design practices because:

- Hidden similarities between different concepts of a similar project can be highlighted,
- Complexity and size of design problems can be lowered.

The present article describes a practical implementation of the dimensional analysis theory in a design context. In addition of the aforementioned expected properties, we expect that dimensional analysis can provide some hints or framework to conveniently combine expected performances of different natures (hereafter named *heterogeneous*) in a preference aggregation model for a global assessment of design concepts (not developed in the present paper). The article itself is organized in the following manner.

The second chapter briefly explains the basic concepts of dimensional analysis [3]. Existing links between background research and dimensional analysis are explained. These links can be the premise for unifying several design theories in the future. A clear summary is made about the theoretical and necessary conditions sufficient for using dimensional analysis in a consistent process of performance modeling and conceptual design assessment. Existing links with other design theories are briefly evoked. Moreover, important aspects related to metrics are introduced in this part.
The concept of exergy [4] is also presented as the result of the use of dimensional analysis on resource consumption and environmental impact accountancy performances. It is shown that this simple metrics is very convenient for embedding various aspects of eco-design at the preliminary design stages.

The third chapter presents the actual industrial case, the preliminary design of a workplace. A workplace is a complex system involving numerous technical, social and economical performances. Moreover, the clients of a given workplace are more and more sensitive to the environmental characteristics of their workplace since they are directly considered as part of the company's environmental performances. Our approach of performance modeling process is applied so as to result in a first step as a set of aggregate performances of the same type. In a second step (only briefly introduced in this article), an overall evaluation or objective function for the whole workplace concept should be analyzed. This second step is not really treated in this article, only the main possible choices related to the method have been highlighted.

The fourth chapter is a synthesis of the main results and it discusses more thoroughly the general implications of this approach for the design process. The results show that it is possible to analyze and model design problems by using dimensional analysis. The insight provided by the dimensional analysis approach in terms of minimization of problem complexity and qualitative modeling of design problems is demonstrated and critically evaluated. The intrinsic ability of the method to highlight similarities between concepts and to capitalize knowledge is also demonstrated and discussed. Future perspectives of this research in terms of multi-criteria optimization [5] are introduced.

2 BASICS OF DIMENSIONAL ANALYSIS AND METRIC SPACE

2.1 Introduction

The purpose of this chapter is to make a basic presentation of some important concepts of the dimensional analysis theory. The dimensional analysis theory as such as been historically developed mainly in order to build similarities between machines or elements of these machines. This tool has been useful to extrapolate results obtained on a prototype to the real machine or process. The effects of this approach has greatly influence physics and numbers of examples of numbers obtained using the dimensional analysis approach can be found in fluid dynamics for example (Froude number, Reynolds number,etc…). Some of the basic ideas of similarity and dimensional analysis had already surfaced in Fourier's work in the nineteenth century's first quarter, but the subject received more methodical attention only toward the close of that century, notably in the works of Lord Rayleigh, Reynolds, Maxwell, and Froude in England, and Carvallo, Vaschy and a number of other scientists and engineers in France. By the 1920's the principles were essentially in place: Buckingham's now ubiquitous p-theorem had appeared (Buckingham, 1914), and Bridgman had published the monograph which still remains the classic in the field (Bridgman, 1922, 1931). Since then, the literature has grown prodigiously [3].
In this section, we summarize the basics of dimensional analysis and we establish a link between dimensional analysis and modern design theories using the formalism of the field of mathematic called topology.

2.2 Basics of dimensional analysis

At the heart of dimensional analysis is the concept of similarity. In physical terms, similarity refers to some equivalence between two things or phenomena that are actually different. For example, under some very particular conditions there is a direct relationship between the forces acting on a full-size boat and those on a small-scale model of it. The question is what those conditions are, and what is the relationship between the forces? Mathematically, similarity refers to a transformation of variables that leads to a reduction in the number of independent variables that specify the problem. Here the question is, what kind of transformation works? Dimensional analysis addresses these questions. Its main utility derives from its ability to contract, or to make more concise, the functional form of physical relationships. A problem that at first looks impossible may sometimes be solved with little effort after dimensional analysis.

Dimensional analysis is the only option to solve problems where the equations and boundary conditions are not completely articulated and it is always useful because it is simple to apply and quick to give insight. To understand its principles, we must return to some of the very fundamental concepts in science. Dimensional analysis is rooted in the nature of the artifices we construct in order to describe the physical world and explain its functioning in quantitative terms.

2.2.1 Physical properties

Science and more specifically design science give a lot of importance to the observation and to the precise description of things and events. But description in absolute terms is impossible and this is the very first step on which dimensional analysis stays. We can do no more than compare one thing with another. When we say that something *is* a bicycle, we mean that it has a set of attributes that are in some way shared by certain objects we have agreed to call bicycles. A physical property is formalized by defining a comparison for determining whether two physical attributes of samples of it are equal (A=B) or unequal (A≠B). Properties of the same kind are also compared. Properties of different types cannot be compared because there is no operation that defines equality. For example it is meaningless to try to compare a particular mass and a particular length because no procedure exists for making the comparison. If a property is defined only in terms of a comparison, we have a procedure for establishing whether two samples of it are equal or unequal, but no concept of what it means for one to be larger or smaller than the other. For example, we know how to determine that two objects have the same shape, or the same color. But there is no sense in asking whether blue is smaller or larger than red. Properties like shape and color are useful for describing things, but cannot play a role in any quantitative analysis, which deals with relative magnitudes. This is why we need to establish comparison operations in design when we want to compare characteristics with each other.

2.2.2 Physical quantities and base quantities

The fundamental goal of science is to deduce laws from observation in the most general manner in order to explain the most possible of the phenomenon. The language of mathematics is ideally suited for expressing laws and it ensures that physical constraints will be followed. A certain limited number of allowed types of properties are accepted in models and they are called physical quantities. There are two types of physical quantities: base quantities and derived quantities. The base quantities form a complete set of basic building blocks for an open-ended system of derived quantities that may be introduced if necessary. The base and derived quantities together provide a rational basis for describing and analyzing the physical world in quantitative terms. A base quantity is defined by specifying *two physical operations*: a comparison operation for determining whether two samples A and B of the property are equal (A=B) or unequal (A≠B) and an additional operation that defines what is meant by the sum C=A+B of two samples of the property. Base quantities with the same comparison and additional operations are of the same kind (i.e. different examples of the same quantity). Quantities with different comparison and additional operations cannot be compared or added. No procedures exist for executing such operations. All physical quantities are *properties* of physical things or events. A base quantity is thus a property for which the following mathematical operations are defined in physical terms: comparison, addition, subtraction, multiplication by a pure number, and division by a pure number. These operations are performed on physical properties of the same kind and the outcome is a physical property of that kind. In addition, each physical operation obeys the same rules as the corresponding mathematical operation for pure numbers.

2.2.3 Units, dimensions, and dimensionless quantities

Units:

The two operations that define a base quantity make it possible to express any such quantity as a multiple of a standard sample of its own kind. The standard sample (i.e. a unit) may be chosen arbitrarily. The comparison allows the replication of the unit, and the operation of addition allows the identification and replication of fractions of the unit. The measuring process consists of *physically* adding replicas of the unit and fractions thereof until the sum equals the quantity being measured. The numerical value of a base quantity depends on the choice of a unit. A physical quantity exists independent of the choice of a unit.

Dimensions and dimensionless quantities:

In order to avoid talking of "units" for quantities that may have no physical representation, but whose numerical values nevertheless depend on the choice of base units, the concept of *dimension* has been introduced. Each type of base quantity has by definition its own dimension. If A is the numerical value of a mass, we write it as $[A]=M$ where the square brackets imply *the dimension of* and M symbolizes the concept of mass. The dimension of any derived physical quantity is a product of powers of the base quantity dimensions. Sums of derived quantities with the same dimension are derived quantities of the same dimension. Products and ratios of derived quantities are also derived quantities, with dimensions which are usually different from the original quantities. All derived quantities with the same dimension change their values by the same factor when the sizes of the base units are changed. A derived quantity is *dimensionless* if its numerical value remains invariant when the base units are changed. The *dimension of a dimensionless quantity is unity*, the factor by which the quantity's numerical value changes when its base units' sizes are changed. Special functions (logarithmic, exponential, trigonometric, etc.) of *dimensional* derived quantities are in general *not* derived quantities because their values do not in general transform like derived quantities when their base unit size changes. Only when the arguments of these functions are *dimensionless s*pecial functions with dimensionless arguments are therefore *derived quantities* with dimension unity.

2.2.4 Dimensional analysis and Vashy-Buckingham's Π-theorem

In a formal way, the Vashy-Buckingham theorem states that: *When a complete relationship between dimensional physical quantities is expressed in dimensionless form, the number of independent quantities that appear in it is reduced from the original n to n-k, where k is the maximum number of the original n that are dimensionally independent."*

In practice, it means that if we imagine a performance Q_0 of a technical system and if this performance is a function of a *complete* set of *independent* parameters Q_1 to Q_n:

$$Q_0 = f\,(Q_1, Q_2, \ldots, Q_n) \qquad (1)$$

Then it is possible to transform this initial set into a reduced set of size k of dimensionless groups having the following form:

$$\Pi_0 = f\,(\Pi_1, \Pi_2, \ldots, \Pi_{n\text{-}k}) \qquad (2)$$

The main interest of this theorem is to diminish the initial size of the problem. Another interest of the approach developed by Bashkar and Nigam [16] is to model relations between dimensionless groups. It is possible to obtain qualitative models of systems. This is due to the facts that dimensional representations of physical variables encode a significant amount of physical knowledge, dimensionless numbers provide a representation of the physical processes, and they can be obtained without direct explicit knowledge of the underlying laws of physics. This approach is used later in the practical case.

2.2.5 Dimensional analysis and design theory

In order to introduce this section, we will first consider a well known design theory called General Design Theory (GDT) [11, 12]. This theory is a notable exception in the set of design theories because the theory is based on mathematical foundations, more specifically on the language developed in general topology [13]. Since its presentation in English already over 20 years ago, GDT has been hardly ever used or even referred to, by researchers other than its developers. Those who have referred to GDT's concepts never used them to guide their work in a practical manner. We want in this article to contribute to a more practical framework for applying GDT's precepts by highlighting that a clear connection exists between GDT (more specifically Axiom 4 of GDT) and the dimensional analysis theory. The analysis of GDT theory has already been done by few authors [14] [9], the readers can refer to these works for a better understanding of the entire GDT approach. In summary, the major point of interest for us in GDT theory is the Axiom 4 which states that a hierarchy of recognition/separation exists between concepts of solutions in design. This axiom states that the best manner to recognize and separate concepts is to analyze the design problem in a topological space called a *metric space*. What is a metric space? The common definition used to define a metric space as follows:
There exists a metric on a topological space S such that a set S is called a *metric space* if with every pair of points x, y $\in$ S, there exists a non-negative real number d(x, y) that satisfies:
If d(x, y) = 0 then x= y and d(x, x)=0,
For any pair of points x, y, d(x, y) = d(y, x),
For any three points x, y and z, d(x, z) $\leq$ d(x, y) + d(y, z).

Where we call a metric space each couple (S, d) where d is a metric on S.
There is numerous examples of metric space but a practical example can be the measure of dimensions of elements in a CAD drawing. To properly use this definition [13], a practical manner to obtain a metric space should be described. The work of Coatanéa [9] has established a list of three necessary conditions to obtain a metric space out of a multi-dimensional design space during the design process. It has been demonstrated that a metric space can be built out of an intermediate topological space called classification space [15]. A classification space is a topological structure resulting from a classification. The first necessary condition states that the design problem should be expressed in the form of a classification space. A classification is a topological structure called classification space [15]
In order to transform a *classification space* into a metric space tree conditions are needed:
These conditions are:
- Having a fundamental system of entourages,
- Having a sufficiently detailed fundamental system of entourage in order to ensure separation,
- Having a countable fundamental system of entourage.

A *fundamental system of entourage* is a set of fundamental concepts used in order to describe things and events. There are major similarities with the description made in section 2.2.1 (i.e. physical properties). A system of entourage integrates for example concepts such as domain of design, functions type, organs, substances and fields, laws [9]. A sufficiently detailed system of entourage means that the description level of the concepts enables recognition between the concepts. A *countable system of entourage* refers to section 2.2.2 because it means that a system of basic quantity is needed for describing and analyzing the physical world in quantitative terms. This system of basic quantities is associated with units. The selected system associating quantity and units is an enhanced SI (International System of Quantities and Units) system [9]. In the enhanced system two quantities have been added, a quantity of cost and a quantity of information.

3 INDUSTRIAL PROBLEM: PRELIMINARY DESIGN OF A WORKPLACE

3.2 Introduction

In this section we introduce the case study of a tertiary workplace (i.e. a place inside which people work in a localized place of a company). Because of the format requirements of this article we have limited our study to the partial analysis of two service functions (i.e. expected actions of a product according to the need of a specified user [6]). These service functions are:

- Improving the satisfaction of the occupants in a physical work environment,
- Reducing the workspace energy consumption level.

We further limit the scope of this example specifically to the visual comfort aspects of the occupant and to the analysis of the energy consumption due to the lighting system.

The study is organized in the following manner. At first, with the help of some specific graphical tools of the value analysis [7] and some other specific graphical tools, the study highlights the technical functions (i.e. the internal actions of a product in order to achieve its service function [8]) that we should fulfill in this study. We list the expected performances linked with these technical functions. We point out performance indicators selected in order to evaluate the performances and consequently the fulfillment level of the technical functions. In a second stage, the interactions and main characteristics are modeled by using a traditional block diagram. Then, the dimensional analysis approach is used in order to model the structure via Π numbers. This stage provides a condensate list of performance indicators and a qualitative model of the lighting and environmental interactions.

The third stage of the analysis consists of using an optimization approach in order to aggregate partially or totally the list of performance indicators and to allow compensation of one performance indicator on the other. This stage is a question of choice because the literature related to multi-objectives optimization is rich and provides several optimization approaches.

3.2 Modeling and analyzing approach

3.2.1 Introduction

Our modeling and analyzing approach is taking into account the formal requirements expressed in chapter 2. This means that the fundamental conditions for metrization should be met in our example. In practice, the first stage of our analysis needs to provide a design problem presented in the form of a classification structure, this classification must be associated with a fundamental system of basic concepts and this fundamental system of basic concepts must be countable. Our analysis is based on a limited number of clearly defined concepts (i.e. function, service function, technical function, performance, performance indicators). The definitions of the concepts are not given in this article but can be found in Coatanéa [9]. The design problem is then classified by using this list of generic concepts. Every indicator is linked with countable metrics. These metrics are derived from a fundamental system of quantities (i.e. the enhanced SI system [9]). The design process can be roughly described as a four step procedure described below. The major tools for analyzing the design process are classical tools of the designer tool box [7] [10]. In this respect our work is integrating existing approaches rather than developing totally new perspectives. The outcomes of these tools are respectively description of:

- The service functions,
- The technical functions or main functions,
- The final refinement of the design problem in the form of a model of flows, variables and performance indicators

When the final refinement of the design problem is obtained, the phase of synthesis of solutions follows. During this phase design solutions are created. This article is not analyzing the design solutions as such. In our case the design solutions are mainly modifying the quantitative and qualitative values of the flows, variables and performance indicators. In this article, we are presenting a partial model of a workplace taking into account only the visual comfort. The problem of resource consumption and environmental impact are not treated but they have been already extensively analyzed in another article [19] by using the concept of exergy.

3.2.2 Expression of the needs

The analysis of the needs is not the purpose of this article. This article is scoped more specifically on building a generic minimized model of a workplace and on the analysis of the multiple performances indicators which needs to be taken into account.

3.2.3 Synthesis table and block diagram

A simplified table, presented partially below, integrates different parameters: - the service functions, the technical functions, the various aspects of the performances of the technical functions, the indicators used to measure the performances, the derived quantity and unit of the indicators and the targets of the indicators. It should be noticed that the technical function studied here consists of promoting visual comfort of a room occupant. Consequently, parameters such as light intensity (Cd) or light power (Cd.sr or lumen) are not included in the table because these parameters are performance indicators of the lamp itself. We have integrated parameters directly measuring the occupant visual comfort. These parameters are defined by the European norms. In this respect, the performances parameters of the lamps are linked with the parameters listed below but they should be seen as causes when the parameters listed below can be understood as consequences.

Table 1: Partial synthesis diagram for the service function ***improving occupant satisfaction***

Service Function	**Technical Function**	**Performances**	**Indicators**	**Qualitative or quantitative**	**Quantity/Unit**	**Target**	**Sources**	**Comments**
SF1: Improve occupant's satisfaction in physical work environment	Promote Visual comfort for the different tasks in workplace	Illumination level in different space	Average Illumination level in different space **IlevelA**	Quantitative	**$Iv.1.L^{-2}$** (cd.sr.m-2) Lux	300 lux 500 lux	EN12464	
		Light uniformity	Light uniformity I_{min}/Iav **Lv**	Quantitative	1	>=0,7	EN12464	
		Luminance distribution of luminaire	Unified Glare Rate (**UGR**) UGR = 8 log (0.25/Lb x L^2/p^2)	Quantitative	**$\log(Iv.L^{-2})$** log (cd.m^{-2})	19 for occupied space 22 for other space	EN12464	
		Balanced luminance in lit environment	Wall reflectance control (Wall, ceiling, floor, Work surfaces) measures to ensure balanced level of luminance **IRef**	Quantitative Qualitative	**Iv/Iv=1** cd/cd=1	60%<wall<90% 30%<ceiling<80% 10%<floor<50% 60%< WS<90%	EN12464	
		Color rendition	Color rendition index (**CRI**)	Quantitative	1	>=80 for common activities >=90 for fine color discrimination	HQE 2006 & EN12464	
		Visual ambiance adapted to work activities	Visual ambiance (Color temperature) **CT**	Quantitative	**T** °K	>=3000 for common activities >=5000 for fine color discrimination activities	HQE 2006 & EN12464	
		occupants control of artificial lighting to suit individual task needs and preferences	% of occupants controlling lighting systems to suit individual task needs and preferences **CAL**	Quantitative	1	90 %	LEED-CI	
		Daylight penetration	Daylight factor I_i/I_O **DP**	Quantitative	1	>=2% for 80% of space	(LEED-CI & HQE)	
		Daylight acess	proportion of occupied space having a daylight access **DA**	Quantitative	1	100% for the occupied offices	HQE 2006 & LEED-CI	75% for occupied LEED-CI
		Outside view	proportion of occupied space having a outside view **Ov**	Quantitative	1	100% for close and open space 40% for other space (meeting room, relax space..)	HQE 2006	75% for occupied space for LEED-CI
		Glare control	Beam angle, mounting angle **BA, MA**	Quantitative	1	avoid glare	HQE 2006	
		individual Contol of daylight penetration	% of occupants controlling daylighting systems **DC**	Quantitative	1	75 %	LEED-CI	

The second stage consists of presenting and extending the information defined above in the form of a block diagram. The goal of this diagram is to give a global technical perspective integrating also the specific technical performances of the lamps, windows, screens. In addition, this block diagram is helping us to model potential interactions between elements pertaining to the visual environment but also between the light environment and the energy consumption level.

Table 2: Partial synthesis diagram for the Service function ***reduce workplace energy consumption***

Service Function	Technical Function	Level of performances	Performances	Indicators	Qualitative or quantitative	Unit	Target	Sources	Comments
SF2: Reduce Workplace energy consumption level	Optimize Energy Performance of Lighting systems	Indoor lighting systems (installed & plug-in lighting systems)	Installed indoor lighting systems	Annual energy consumption of lighting systems	quantitative	$M.L^2.T^{-2}$ kg m^2s^{-2} kwh/year			
			Lighting power density	The maximum lighting power per m^2 of a building classification of space function.	Quantitative	M.T-3 kg s-3 kwh/m2	Reduce lighting power density betwwen 15%-25% below the standard	LEED-CI	
			Installed Baseline Lighting Energy	Installed Baseline Lighting Energy power	Quantitative	$M.L^2.T^{-3}$ kg m^2s^{-3} Kwh			
	Provide Lighting Controls in the workplace	implementation of Lighting controls to reduce energy use	Lighting Control systems typology	Lighting management to conserv lighting energy use	Qualitative	low/medium/good	Good level	LEED-CI	most manufacturers provide estimations of potential saving
		potential energy savings	Potential Energy savings by type of control system	Estimation of Energy savings	Quantitative	1			

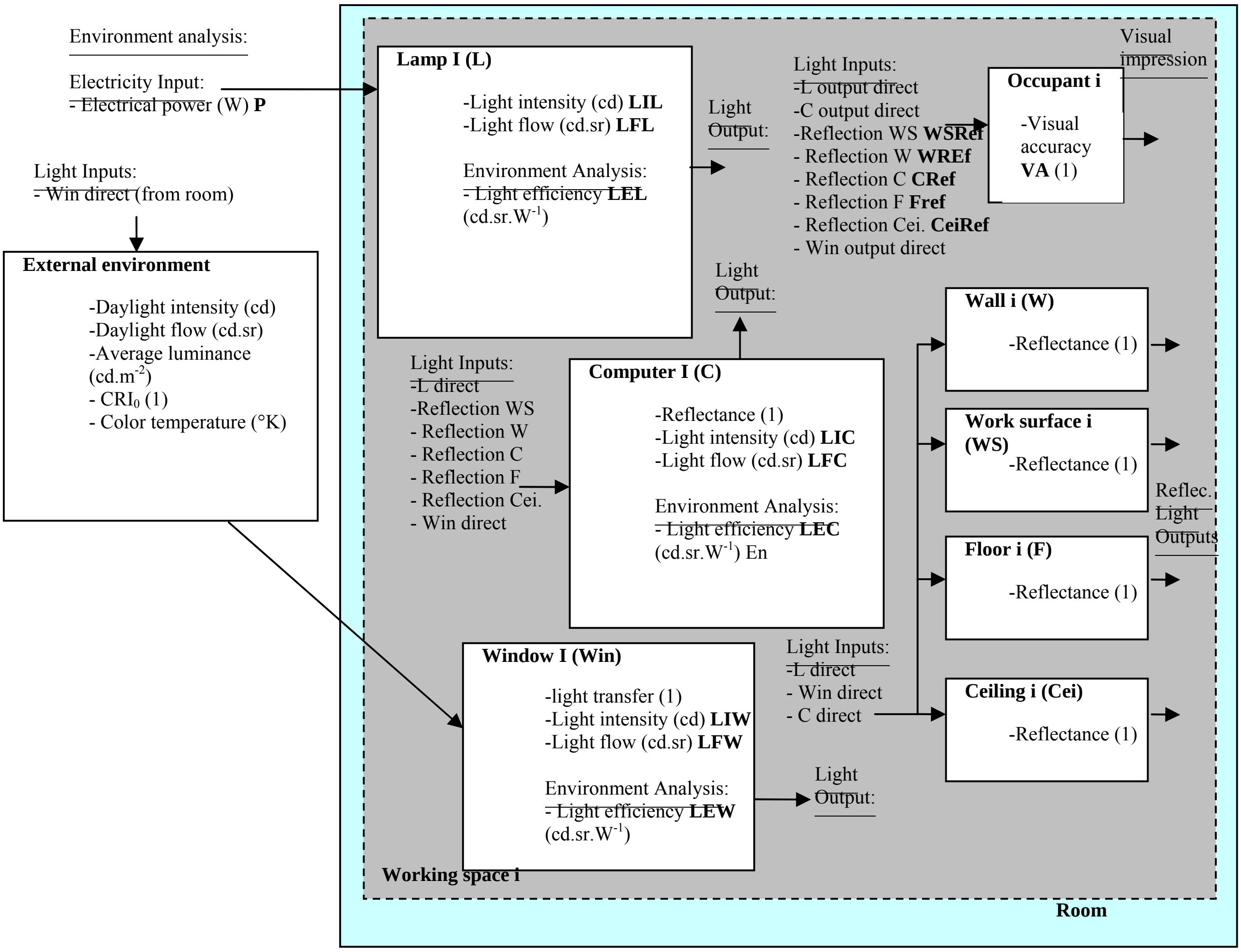


Figure 1: Partial block diagram of the light interactions in the room

3.2.4 Transformation of the design problem by using dimensional analysis

Figure 1 presents a partial synthesis of some performance indicators not listed in Table 1. In addition, this graph provides a decomposition model of the problem where clusters can be identified. These clusters can now be analyzed by using the mathematical machinery developed by Bashkar and Nigam

[16] in order to model problems via dimensionless numbers. A short summary of the approach is needed because this aspect of the work represents the main contribution of our article in our viewpoint. At first, if the necessary fundamental conditions are met (see section 2.2.5), dimensionless groups may be created. Dimensionless groups can be made by using two different approaches. The first one that we propose to call *top-down approach* is analyzing the overall workspace by listing all the performance variables. The Vashy-Buckingham theorem is then used directly on this entire set of variables. This method has a major drawback; it blurs the qualitative physical meaning of the dimensionless groups by mixing attributes belonging to different parts of the system. A second approach presented by Tomiyama [17] that we propose to call *bottom-up approach* consists of creating dimensionless numbers at the organ level and then aggregating them. This approach is better because the physical meaning of the dimensionless groups is kept. The approach presented here is a mix of both approaches because the Vashy-Buckingham theorem is used inside our clusters to create dimensionless numbers whereas Tomiyama's does not use it.
How to create in practice dimensionless groups for the cluster called *Occupant i*. The overall performance for the occupant, the *visual comfort* is according to our analysis depending on 24 different parameters and can be written in the following manner.

$$Visual_{comfort} = f(I_{level\,Av}, L_U, UGR, W_{\mathrm{Ref}}, Cei_{\mathrm{Ref}}, WS_{\mathrm{Ref}}, F_{\mathrm{Ref}}, C_{\mathrm{Ref}}, CRI, CT, CAL, DP, DA, OV, BA, MA, DC, VA, P, LIL, LFL, LEL, LIC, LFC, LEC, LIW, LFW, LEW) \tag{3}$$

Minimization of the design space by using the Vashy-Buckingham theorem (algorithm of Butterfield [18]):
The fundamental dimensions of our case are: Iv (luminous intensity) L, T
These three dimensions can be organized in a table (see Table3). The table clusters the variables according to table 3 [18].
Where:

- *V* is the list of the independent variables which are assumed to govern the system,
- $R \in V$ contains the variables selected from V, which have distinct dimensions other than 0,
- *P* are variables not in R which have been placed in this group because the dimensions of some of these variables repeat the dimension of the variables in R.
- *O* are variables which have the dimension 1,
- *D* is a possible set of *m* independent variables from basic or composed dimensions.
- *Q* is a set of variables selected from *R*, from which a dimensionless group cannot be formed. *Q* list is the *repeated variable* list.

Table 3: Table for the selection of the repeating and performance variables (adapted from Butterfield [18])

<table>
<tr><td colspan="2"></td><td colspan="9">V</td></tr>
<tr><td colspan="2"></td><td colspan="5">R</td><td colspan="2" rowspan="2">P</td><td colspan="2" rowspan="2">O</td></tr>
<tr><td colspan="2"></td><td colspan="3">Q</td><td colspan="2">S</td></tr>
<tr><td colspan="2"></td><td>v₁</td><td></td><td>v_m</td><td></td><td>v₀</td><td></td><td>v_p</td><td></td><td>v_n</td></tr>
<tr><td rowspan="5">D</td><td>d₁</td><td colspan="3" rowspan="5">A (mxm)</td><td colspan="6" rowspan="5">B(mx(n-m))</td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td>d_m</td></tr>
</table>

The array (*mxm*) [A] is the outcome of the process of selection of the variables. In order to be able to form dimensionless numbers, it should be checked that [A] is non-singular (det (A)≠0).
Then it is necessary that:

- No column of [A] contains entirely zero elements,

- No column of [A] is either repeated or a multiple of another one,
- The column of [A] cannot be combined to form a zero column. This requirement is similar to the selection of the variables of *Q* in order to avoid that they can form a *dimensionless group*.

All these conditions are similar to say that the rank of [A] is *m*. This is the condition that defines the number of components of *D* to be D_{min}. The list *Q* is often not unique.
The analysis of the completeness of the list of variables is out of the scope of this article. A contribution to this aspect of dimensional analysis is made in Coatanéa pp.181-189 [9].
The matrix for our design problem is given in Table 4.

Table 4: Table for the analysis of the visual comfort of the occupant

		V																								
		R																								
		Q		S	P							O														
		I_{levelA}	*LIL*		*LFL*	*LIC*	*LFC*	*LIW*	*LFW*	CT	e^{UGR}	Lu	Wref	Ceiref	Wsref	Fref	Cref	CRI	CAL	DP	DA	OV	BA	MA	DC	VA
Dimensions	L	2	0		1	0	1	0	1	0	2															
	T	0	0		0	0	0	0	0	1	0															
	Iv	1	1		1	1	1	1	1	0	1															

According to this matrix, it is possible in the best case to create 7 new dimensionless numbers. Nevertheless, in our design case, we have 15 variables already having a dimension 1. It should be noticed that the choices of the parameters present in Q are not unique. In addition, it is not possible by using the variables that we have in our system to find a dimensionless number associated with this variable. The color temperature *CT* and the color rendition index *CRI* are both linked. A new index is under development because the *CRI* has come under a fair bit of criticism in recent years. The 6 new dimensionless numbers are expressed through the following formulas:

$$\Pi_1 = LFL.{I_{levelA}}^{-1/2}.LIL^{-1/2} \tag{4}$$

$$\Pi_2 = LIC.LIL \tag{5}$$

$$\Pi_3 = LFC.{I_{levelA}}^{-1/2}.LIL^{-1/2} \tag{6}$$

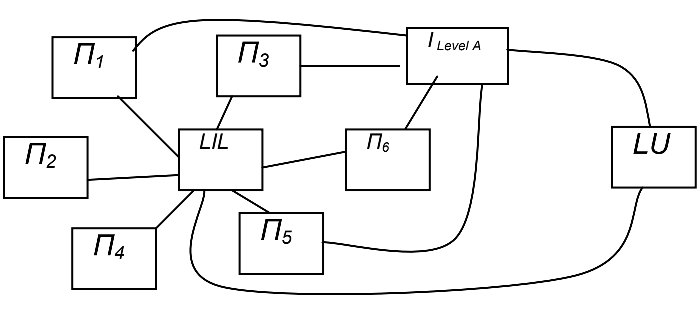
$$\Pi_4 = LIW.LIL \tag{7}$$

$$\Pi_5 = LFW.{I_{levelA}}^{-1/2}.LIL^{-1/2} \tag{8}$$

$$\Pi_6 = e^{UGR}.{I_{levelA}}^{-1/2}.LIL^{-1/2} \tag{9}$$

A general graph (partially represented in Figure 2), combines those numbers with the 15 numbers of the group *O*. The Equation 3 is minimized and takes the form of the Equation 10.

Figure 2: General graph of the visual environment (partial representation)



$$Visual_{comfort} = f(CT, \Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5, \Pi_6, LU, W_{Ref}, Cei_{Ref}, WS_{Ref}, F_{Ref}, C_{Ref}, CRI, CAL, DP, DA, OV, BA, MA, DC, VA,) \tag{10}$$

The design model related to the visual comfort has been minimized but the structure is not the one of a metric space because CT has another dimension than the dimension 1 (i.e. T).

Model of the interaction by using the qualitative approach (Bashkar and Nigam [16]):

The approach of Bashkar and Nigam states than a machinery can be built (i.e. a machinery aimed at modeling the interaction of the graph presented in Figure 2) in order to model *intra* and *inter Interactions* within dimensionless groups and clusters. Interactions can take place: within a dimensionless group between the attributes; across dimensionless groups, if they are connected through *a contact attribute* and finally across functions via a *coupling dimensionless group* [16].
In our case, to demonstrate the machinery we can compute the *intra-dimensionless group* interactions partial of the dimensionless group Π_1:

$$\Pi_1 partial_1 : \frac{\partial LFL}{\partial I_{levelA}} = \frac{1}{2}\frac{LFL}{I_{levelA}} \rangle 0 \qquad \Pi_1 partial_2 : \frac{\partial LFL}{\partial LIL} = \frac{1}{2}\frac{LFL}{LIL} \rangle 0 \tag{11}$$

The meaning of this computation is to show that if the Average illumination level I_{levelA} is increasing by a value ΔI_{levelA} then the light flow coming from a lamp, from the windows or from the computer has increased. In a the same way by using the results of the *partial 2*, if the light intensity has increased then the light flow should have increased too. It is also possible to built *partials of inter dimensionless groups*.
There is for example in Equations 4 to 9, 2 contact variables (LIL and I_{levelA}). These variables act as interacting variables between the dimensionless numbers 1 to 6. Thus, one inter-regime partial of groups Π_1 and Π_2 is:

$$\left[\frac{\partial LFL}{\partial LIC}\right]^{LIL} = -\frac{1}{2}\frac{LFL}{LIC} \langle 0 \tag{12}$$

In this section, we have tried to show that the design model of our problem can be minimized by using dimensional analysis. We have also demonstrated, even if only partially, that a mathematical machinery can be built in order to model interrelation inside our model. These interrelations can provide interesting insight in analyzing the consequences of different changes in the performances. Nevertheless in this model we have managed to relate the performance indicators together but no formal link has been established between the performances indicators and the technical function goal of our assessment.
In order to achieve this goal a multi-objective optimization stage is necessary. The π numbers computed in this article is forming a type of partial aggregation of the design criteria. The attributes are automatically weighted and form a global coherent evaluation and comparison framework. The multi-objective perspective of the dimensional analysis approach has been voluntarily omitted in the present article for reason of brevity. Nevertheless, our approach is of particular interest for multi-objective optimization and can be fruitfully combined with methods such as the evaluation method developed by Professor Ashby [20].

4 DISCUSSION AND CONCLUSION

This article has tried at first to highlight fundamental similarities between dimensional analysis and axiom 4 of GDT. Fundamental principles in order to transform an initial design space into a minimized one have been depicted. Then a practical example related to the design of a workplace improving the satisfaction of the occupants has been presented. In this example, the visual comfort of the occupant has been modeled and the initial model has been refined according to the theoretical basis presented in the first part. A formal model based on dimensionless numbers has been presented. This model is seen by the authors as an initial step in the analysis but it should be followed by a multi-objective optimization where overall goals are aggregated and linked with the performance indicators. We consider this article as a first exemplified attempt which will be soon developed and completed further in future researches, notably to provide a more complete framework of design selection in the preliminary design stages.

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