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GEOMETRIC AND TOPOLOGICAL MODELLING OF 3D CRUMPLED STRUCTURES

Serge ROHMER (1), Alexis MERAT (1), Vincent FLODERER (2), Alain COPINET (3)
1: University of Technology of Troyes, France; 2: International Research Center on Modelling with Paperfolding, France; 3: Engineer School of Reims, France

ABSTRACT

Crumpling is a new method derived of the origami techniques that transforms a single paper sheet into a three-dimensional structure by a systematic generation of random folds. The resulting crumpled object is an innovative answer to packaging industries that need attractive and dynamic products in a sustainable context. In order to understand the performances of crumpled structures and their applications in the field of packaging, this paper firstly details the fundamental characteristics of crumpled surfaces by expressing categories of crumpled creases patterns and patterns networks. The crumpled surfaces connectivity is then studied in an Extended Attributed Adjacency Graph by adding new faces and edges attributes.

Keywords: structural modelling, graph based, crumpling, eco-design

Contact:
Dr. Serge Rohmer
University of Technologie of Troyes
Charles Delaunay Institute
TROYES
10004
France
serge.rohmer@utt.fr

1 INTRODUCTION

Industries have been using packages for a long time for different purposes. They can protect products from contamination or environmental damage, facilitate transportation and storing of all kinds of products, or carry information. Depending on their applications, they are described as flexible, semi-flexible, or rigid. In terms of environmental performances, the report in (Flexible Packaging Europe, 2011) considers that a flexible packaging is the most source-reduced form of packaging. It contains the least amount of material, and consequently adds little weight to the product, and reduces the waste quantity for an optimized recycling. By comparison to rigid packaging which essentially exploit planar and basic curved faces and are considered as static objects, flexible or semi-flexible packages permit complex shapes for mass product markets**.** HEXA POT™ for example, is an ultra-light, and 100% biodegradable cookware (Energia USA, 2012), the product must be unfolded and expanded to make the hexagon pot shape. The ice bucket created by the designer Mathias van de Valle is a foldable package that can be easily transported and stored before and after use. These packaging exploit the advantages of origami principles (Yoshizawa, 1954) to design attractive objects in a competitive market.

To accompany this trend, a new folding method based on crumpling sequences can offer new opportunities to achieve the functions of packages (protect, group, store, transport, inform and seduce) in a sustainable way. Crumpling consists in creating a network of complex 3D crease patterns according to a folding process that generates random folds from a single flexible sheet. Inspired by (Jackson, 2011), crumpling has been used since over fifteen years by the CRIMP team to design a wide range of biomimetic models (Floderer, 2007)(Floderer, 2008)(Mérat et al, 2010). The key idea is that crumpled structures bring powerful mechanical and environmental performances to create dynamic three-dimensional structures. The expected performances of such structures are made possible by eliminating cut and glue processes, as well as the use of a single raw material for a simplified recycling. However, in order to implement such new technique for mass product markets in the packaging sector, researches on crumpled structures must be engaged. It concerns structural 3D modelling, behaviour modelling or simulation under mechanical solicitations, or the modelling of the folding process which generates crumpled crease patterns. These researches can be inspired by those made on rigid packaging that are largely covered by researchers. Finite element framework can understand and predict the behavior of a material during the folding process (Beex and Peerlings, 2009), and mathematical description of elastic membrane can predict the development of ridges and vertices (Wood, 2002). Experimental studies can explain the propagation of failures which affect cracking in a coated layer (Kim et al, 2010). Other researches propose efficient algorithms to give best ways to fold tray cartons (Mullineux and Matthews, 2010), or investigate computational complexity about geometric folding and unfolding (Demaine and O'Rourke, 2005). Some of them use graph based methods (Liu and Taï, 2007)(Ida and Takahashi, 2010) to describe geometric and topological attributes for the understanding of the folding process in terms of operational sequences.

In order to initiate knowledge on crumpled structures, this paper proposes to explore the modelling of their geometric and topological characteristics. Section 2 provides key definitions on fold, crease pattern and crease pattern network applied on crumpled structures. To support the geometric and topological modelling, a graph based method is proposed in Section 3. A revised Extended Attributed Adjacency Graph expresses new attributes that are more adapted for crumpled structures. The graph based model is then applied and illustrated with some examples, and a discussion on its limitations defines some perspectives. Future works are consequently proposed in the conclusion.

2 DEFINITIONS

As previously defined, crumpling is a method derived of the origami folding techniques. The folds are a physical consequence of the containment of a flexible sheet in a restricted volume, as for example the crumpling of a paper ball in the hand. The genesis of fold into crumpled structure is detailed in the following section.

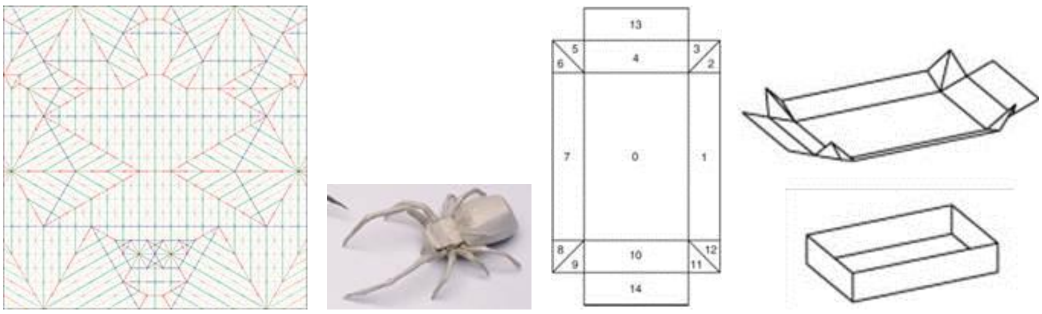
2.1 From fold to creases pattern

A Fold can be viewed as a physical or abstractive entity by philosophers, mathematicians or physicians. Huffman (1976) first explored issues of curvature definition, convexity and concavity, Demaine et al (2000) described methods for constructing arbitrary flat origami silhouette, or how wrap spheres with flat paper (Demaine et al, 2009). In computational geometry, a survey of folding and unfolding can be found in (Demaine and O'Rourke, 2005). Other terminology is proposed in the study of orogenic belts that contain characteristic fold systems. The associated topological terminology often draws a fold as a part of a cylindrical structure containing depression or culmination with crest, trough, inflection point, hinge and limb (Burg and Ford, 1997).

The ontology used in this paper for the modelling of crumpled structures refers to geometric entities used in CAD namely face, edge, vertices and their mutual relationship. Depending also on the application in the field of packaging, the vocabulary uses part of the glossary of printing terms and origami alphabet. Concerning the modelling hypothesis, a 3D crumpled structure is considered as a non-cuted structure erected from a convex 2D flexible sheet. The aggregation level describes the structure of a crumpled object until the crease pattern organisation.

- Fold – a fold is a sharp break or bend in paper sheet that has been bended over itself so that one part of the sheet lies on over another part. A fold changes a surface considered as continue and smooth into a discontinue surface (C^1continuity). A fold is the elementary component of a crumpled surface. In printing terms Half-fold results of a sheet folded in half.
- Crease pattern – a set of folds in a delimited area create a pattern. The physical configuration of folds located in the area represents a crease pattern**.**

The below figures represent two examples of crease patterns. Figure 1a is an origami pattern of a brown widow model developed by the designer Robert Lang and Figure 1c is a pattern (flat layout) of a tray carton (Mullineux and Matthews, 2010). These crease patterns networks are based on classical origami techniques, they describe flat layouts with a low density of creases, and most of the resulting objects are static in term of elasticity.



(a) *(b)* *(c)* *(d)*

Figure 1.Crease pattern (a) of a brown widow (b). Crease pattern (c) of a tray carton (d)

2.2 Crumpled surfaces

Crumpling is a particular configuration of a crease pattern, it generates random creases which are not necessary joined unlike those contained in a classic origami. This distinctive characteristic brings elasticity properties that have been early used for wedging and cushioning applications. Many patents illustrated for example by Levine et al (1992) or Wetsh and Tegel (2011) describe mechanical processes with rolls to transform flat paper into cushions, they have been generating a prosperous industry of paper cushioning machines. The resulting cushions contain random or parallel creases. New investigations based on empirical experiences performed by CRIMP show that crumpled folds networks fall into three main categories: random, parallel, radial.

- Random crumpling – considering a delimited area in a paper sheet, if the direction of the present creases in the area doesn't follow a specific distribution, the pattern is considered as random.
- Parallel crumpling – considering a delimited area in a paper sheet, if the present creases in the area follow a linear direction, the pattern is considered as parallel.
- Radial crumpling – considering a delimited area in a paper sheet, if the present creases in the area follow a concentric direction towards an origin point (inside or outside the area), the pattern is considered as radial.

The attributes associated to the crumpling categories can be formalized by the quadruplet (distribution, origin point, density, geometry) detailed in Table 1. The distribution refers to the organization of the creases into three sub-categories; the origin point refers to the location towards which a radial pattern points (two sub-categories); density refers to the progression mode of creases; geometry refers to the form of the surface. Depending on the aggregation level previously defined, the geometry describes the first order approximation of the surface shape (Figure 2f) into four sub-categories.

Table 1.Geometric attributes of crumpled creases pattern

<table>
<tr><td>Attributes</td><td colspan="2">Sub-categories</td><td>Description</td></tr>
<tr><td rowspan="3">Distribution</td><td colspan="2">Parallel</td><td>creases are parallel (Figure 2a)</td></tr>
<tr><td colspan="2">Radial</td><td>creases point towards an origin point (Figure 2b)</td></tr>
<tr><td colspan="2">Random</td><td>creases do not follow a specific orientation</td></tr>
<tr><td rowspan="3">Origin point</td><td rowspan="2">In</td><td>Mount</td><td>origin point is located inside the pattern (Figure 2b) and creates a pick (Figure 2e)</td></tr>
<tr><td>Valley</td><td>origin point is located inside the pattern (Figure 2b) and creates a cavity (Figure 2d)</td></tr>
<tr><td colspan="2">Out</td><td>origin point is located outside the pattern (Figure 3c)</td></tr>
<tr><td>Density</td><td colspan="2">/</td><td>progression mode of the creases</td></tr>
<tr><td rowspan="4">Geometry</td><td colspan="2">Planar</td><td>the surface that delimits the crease pattern has a planar geometry</td></tr>
<tr><td colspan="2">Convex</td><td>the surface that delimits the crease pattern has a convex geometry</td></tr>
<tr><td colspan="2">Concave</td><td>the surface that delimits the crease pattern has a concave geometry</td></tr>
<tr><td colspan="2">Mixed</td><td>a surface that combines planar/convex/concave geometries</td></tr>
</table>

(a) (b) (c) (d) (e) (f)

Figure 2.Crease patterns of crumpled Kraft paper. First order approximation (f)

By applying the sub-categories of the attributes, the quadruplet (Distribution, Origin Point, Density, Geometry) can take the following elementary configurations based on the CRIMP empirical experiences:

- (Parallel, /, Density, Convex)
- (Parallel, /, Density, Concave)
- (Parallel, /, Density, Mixed)
- (Radial, In/Mount, Density, Mixed)
- (Radial, Out, Density, Convex)
- (Radial, Out, Density, Concave)
- (Radial, Out, Density, Mixed)

Those configurations have been tested and applied on numerous crumpled models developed by CRIMP, as described in the following section. From a practical view point, CRIMP created prototypes of crumpling machines to generate parallel and radial creases. These machines are manually powered, and can serve as the basis for the development of automated machines for mass products markets.

2.3 Crease patterns network

The elasticity property resulting from crumpled folds allow the bending of a network of adjacent crease patterns into three-dimensional configurations.

- Crease Pattern network – a global pattern for a given crumpled object results of the connectivity of crease patterns created from a single paper sheet. The area of each crease pattern are delimited by ridges namely edges in CAD

A network of crease patterns can be characterized by three attributes: distribution, origin pattern, density. A distribution of crease patterns can be parallel, centered or mixed. A parallel distribution

means that the crease patterns are arranged in parallel, a centered distribution contains patterns centered toward a reference pattern. The reference pattern defines the pattern from which a progression can be related. The density expresses the progression mode of the crease patterns, it can be regular as illustrated in Figure 4a or progressive.
A necessary condition to erect a complex 3D structure from an initial crease patterns network (Figure 3a) is the existence of a minimal creases density that brings elasticity property to the model. This mechanical performance allows a dimensional adaptability of the patterns network as shown in the transformation of a folded crease patterns network shown in Figure 3b into an erected crumpled object (Figure 3c), or the transformation of the folded network (Figure 3d) into a 3D object (Figure 3e). The experiments performed for years by the CRIMP members prove that it is possible to create numerous complex three-dimensional structures made of different associations of crease patterns.

(a) (b) (c) (d) (e)

Figure 3.Crumpled Kraft papers. Regular network of (Radial,In/Valley, D, Mixed) patterns (a), CRIMP © 2012. Folded network(b) and its associated unfolded crumpled object (c). Folded network (d) and its associated unfolded crumpled object (e)

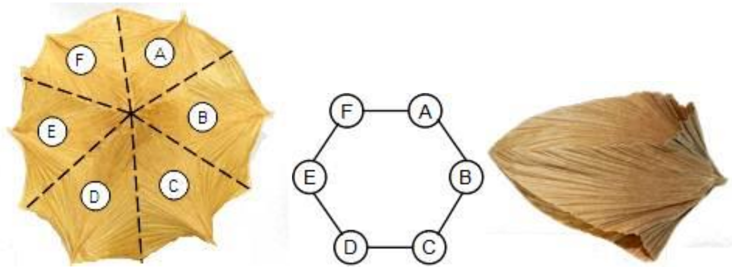
The specific crumpling techniques developed by CRIMP create organized crease patterns very similar to those observed in nature, allowing attractive dynamic objects for packaging applications. In a sustainable context, the expected environmental performances depends on the elasticity property that provides a simplified transportation in a minimum of space (Figure 3d), it also depends on the crumpling process that needs no gluing or cutting activities, and also from the agro-resources that constitute the raw material of the paper sheet for a simplified recycling.

3 GRAPH BASED METHOD

3.1 Face adjacency graph and sub-graphs

Topological information of folded or machined structure with a graph based method is a well know approach for feature recognition. In the domain of computer aided design, the Face Adjacency Graph (FAG) proposed by Ansaldi et al (1985) describes the connectivity among the faces that constitute a 3D structure. The nodes of the graph represent the faces while the arcs between two nodes represent the connection between the corresponding two nodes (faces). Figure 4a illustrates a crease patterns network and its associated FAG (Figure 4b) before to become a crumpled object (Figure 4c). In order to add geometric characteristics of the edges, Joshi and Chang (1988) proposed an Attributed Adjacency Graph (AAG). The nodes still represent faces, arcs represent edges, and the attribute of the arcs is a boolean variable that represents the edge convexity between two planar faces. Later, Gao (1998) developed the Extended Attributed Adjacency Graph (EAAG)in order to express the automatic recognition of machining features. The graph defines edges' attributes (convexity, existence, loop type) and faces' attributes (source, convex hull, number of loops, split status, geometry). Other additional sub-graphs like the Manufacturing Face Adjacency Graph (MFAG) developed by Gao (1998) allow some simplifications. More recently, Lockett and Guenov (2005) defined an Attributed Mid Surface Adjacency Graph (AMAG) where a mid-surface is a dimensionally reduced representation (walls are modelled with surfaces with zero thickness) to simplify the feature recognition for injected plastic objects.
The main difficulty in exploiting a FAG is that the same graph can be associated to several 3D structures. In this case, the unfolding problem requires the transformation of the graph into spanning trees from where constraint-based algorithms allow the selection of optimized flat layouts (Liu and Tai, 2007). Another transformation, performed by Mullineux and Matthews (2010) in his constraint-based simulation of a carton folding operations, allows the reduction of the face graph to a spanning

tree that imposes a hierarchy on the faces and allows the required driving motions for the industrial erection process.



(a) (b) (c)

Figure 4.Centered network of (Radial, In/Mount, D, Mixed) patterns (a), associated FAG (b), transformation of the patterns network into a 3D object (c)

A first analysis is that graph-based methods are useful for feature recognition of objects defined for CAD, the topological and geometric information are structured in a B-rep model data. Moreover, the use of spanning trees derived from FAG is powerful for unfolding problems, as for example a constraint-based simulation that allows the choice of the best flat layout by eliminating interferences or overlapped faces. By considering the crumpling context, EAAG can be considered as the most appropriate graph for crumpled objects. The above section studies the adequacy of the graph within the geometric characteristics of crumpled objects.

3.2 Revised EAAG applied on crumpled objects

The reviewing of the EAAG attributes shows that some changes must be taken into account to fit the crumpling context for the edges' and the surfaces' attributes.

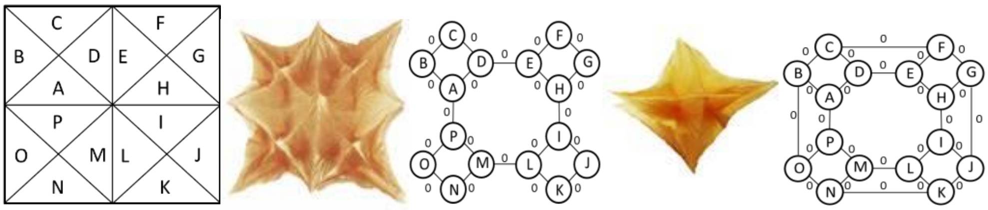
The convexity attribute must be related to the first order approximation for the crumpled edges in order to define the global shape of the edge. The existence attribute (real and virtual) initially dedicated for CAD must be changed because all edges of a crumpled object are already in the real stock product; the terms machined and non-machined are more appropriated to express the crumpling transformation. The loop attribute doesn't change.

For the face characterisation, the current source attribute cannot be applied because our hypothesis is that the final object is fully crumpled, in other words all planar faces are machined. The convex hull attribute should here refer to the first order approximation of the crumpled surface. The quantity of loops can only be applied if a crumpled object is made of several crumpled parts. The split status is not applicable because a crumpled object is made of non-planar faces, it can be replaced by an overlap status that express is a face overlaps another one (status =1) or not (status = 0). Finally, the geometry will refer to our four sub-categories.

In order to illustrate the revised EAAG, Figure 5explains the transformation of a flat layout into two crumpled objects. In this example, Figure 5a gives the initial flat layout from which a network of regular (Radial, In/Valley, D, Mixed) patterns is created (Figure 5b). The associated AAG (Figure 5c) shows that all edges are concave (status=0). Thanks to the elasticity property given to the sheet by the crumpling process, Figure 5d shows another configuration of the object created by the assembly of the peripheral edges. The associated AAG (Figure 5e) shows arcs expressing the new adjacency between faces G and J, K and N, O and B, and finally C and F. For this example, there are no overlapped faces, the 3D object is a manifold structure. In the case of crumpled objects that contain overlapped faces (Figure 3e and 4c), some limitations will be expressed in the next section.

3.3 Discussion

The packaging sector needs regular geometries in order to contain or group products, it justify the use of graph based methods for the modelling of packages. The revised EAAG is then useful for the face adjacency representation of a manifold object made of a structured patterns network with convex and concave faces. However, crumpled objects are not limited to regular geometries, more complex structures have been designed by CRIMP for artistic purposes (exhibit, decoration). These structures will be soon used to assess the revised EAAG.



(a) (b) (c) (d) (e)

Figure 5.Transformation of a flat layout (a) into 2 crumpled objects (b) and (d), and their respective associated AAG (c) and (e). Origami models, CRIMP © 2012

Another limit can be highlighted by comparing the final EAAG with the initial FAG of a crumpled object. It shows that the topological representation is necessary but not sufficient to understand the folding process in terms of successive folding tasks. A hypergraph representation can be useful to solve this problem. Indeed, a hypergraph is a generalization of a graph, in which edges can connect any number of vertices. This property has been explored by Ida and Takahashi (2010) for flat origami, it must be adapted for 3D structures. Crumpling mixes both classic origami tasks from Ida and Takahashi (2010) point of view for the edges creation, and machined tasks from Gao and Shah (1998) point of view for the creases pattern generation. Empirical experiences performed by the CRIMP members during fifteen years shows that a crumpling process uses three fundamental actions that are: concentric pressure, uni-directional pressure, and reversal of surface. A concentric pressure generates a radial pattern, the uni-directional pressure generates a parallel pattern, and the reversal of surface changes a mount surface into a valley surface and conversely. The reversal of surface often follows a concentric pressure in a repeated manner in order to create the crease patterns network. This process increases the density of folds and generates a mechanical hanging among them that confers strength and elasticity to the three-dimensional structure as shown in Figure 4a. Moreover, the reversal of surface can be used to overlap faces. The overlapping process creates closing keys that gives strength and keep the structure in a three-dimensional configuration, as for the tray carton example (Figure 1b) where the face 13 overlaps the face 4, and the face 14 overlap the face 10. The main advantage of a crumpled object is that even after the overlapping process, the structure still keeps elasticity (Figure 3d and 3e). In summary, the three fundamental crumpling actions (concentric pressure, uni-directional pressure, reversal of surface) must be integrated in a new version of a labelled hypergraph. It has been already engaged to express the 3D transformation of the adjacency and superposition relations during the crumpling process. The hypergraph will represent the crumpling process and will serve at the definition of industrial procedures to generate crumpled packaging.

4 CONCLUSION

A geometric description of a new generation of origami based on crumpled paper has been proposed in this paper. The associated geometric characteristics allow a better understanding of their topological modelling thanks to a revised Extended Attributed Adjacency Graph. New faces attributes and edges attributes has been defined in order to highlight the ability of crumpled faces to generate various complex pattern that can be used for attractive packaging. The limits of such graph also shows that in order to understand the crumpling process or the adjacency and superposition relationships between 3D crumpled faces, a new modelling based on the hypergraph concept is useful and will be investigated in a future work**.**

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